Repeating fraction

Property

The repeating fraction =

Proof

Method 1:

By sum of geometric series, one has that

=

=

=

=

=

Method 2:

Let = ( times after the floating point)

The following holds:

for every positive integer .

Thus,

Now, suppose there exist a number such that

One can imply that

=>

For large , approximates to . While for as inf, the limitation of approximates to zero. I.e.

which imply that

as

Hence , .

Therefore, there are only 1 possible when the above make the above implication true (and there are no flaw in the above implication).

as

which is equivalent to

which completes the proof.

Ref

[0.999... - Wikipedia](https://en.wikipedia.org/wiki/0.999...)